Model of contact population dynamics with happy island

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Abstract

We consider the model of branching random walk (contact population model) on multi-dimension lattice $\mathbb{Z}^d$, with the following assumptions:

- individuals move independently of one another on lattice as random walks (defined by some generator $\mathcal{L}$);
- individuals can split into to offspring (duplicate) independently of one another only at the sites $x \in Q$, where $Q \subset \mathbb{Z}^d$ – some finite set, which can be interpreted as happy island), with large constant birth rate Λ;
- at $x \notin Q$ individuals can die independently of one another with constant mortality rate μ.

We consider the super-critical case of this model, i.e. when the operator:

$$(\mathcal{H}\phi)(x) = (\mathcal{L}\phi)(x) - \mu\phi(x) + \Lambda\mathbb{I}\{x \in Q\}\phi(x). \tag{1}$$

has positive eigenvalue $\lambda_\circ > 0$. In this situation the subpopulation, i.e. particle field generated by initial particle at site x:

$$\{n(t,x,y),\ y \in \mathbb{Z}^d,\, t \geq 0\},\ n(0,x,y) = \delta(x-y), \tag{2}$$

increases exponentially fast:

$$\frac{n(t,x,y)}{e^{\lambda_\circ t}} \xrightarrow[t\to\infty]{L_2} \xi_x^*(y) \tag{3}$$

where limit field $\{\xi_x^*(y),\ y \in \mathbb{Z}^d\}$ uniquely defined by its factorial moments

$$M_{l,x}(y) = \mathbf{E}\xi_x^*(y)\,(\xi_x^*(y)-1)\cdot\ldots\cdot(\xi_x^*(y)-l+1) \tag{4}$$

satisfied Carleman conditions.

We consider ecological front of subpopulation (2) which can be defined as

$$\Gamma_t = \{y = y(t) : \mathbf{E}n(t,x,y) = O(1)\}, \ t \to \infty, \tag{5}$$

and we show that there is no intermittency inside this front.

We consider the probability of vanishing of the total subpopulation

$$q(x) = \mathbf{P}\left(\sum_{y\in\mathbb{Z}^d} n(t,x,y) \xrightarrow[t\to\infty]{} 0\right).$$