

# On Vlasov type Equations: derivation, hydrodynamic and Hamilton-Jacobi consequences and cosmological solutions.

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Now there are exist Vlasov-Poisson equations, Vlasov-Maxwell equations, Vlasov-Einstein equations, the names were introduced mainly by French mathematicians (Choquet-Brua, etc., see [1-15]), but have become generally accepted. The story of Vlasov-type equations will be presented. In classical textbooks (Pauli; Fock,; Landau and Lifshitz; Dubrovin, Novikov, Fomenko; Weinberg; Vlasov ...), equations for fields in the Einstein and Maxwell equations are proposed without deducing the right parts. Here we give the derivation of the right-hand sides of the Maxwell and Einstein equations within the framework of the Vlasov-Maxwell-Einstein equations from the classical, but slightly more general principle of least action. A method of transition from kinetic equations to hydrodynamic consequences is proposed, as it was done earlier by A.A.Vlasov himself. In the case of Hamiltonian mechanics, a transition from the hydrodynamic consequences of the Liouville equation to the Hamilton-Jacobi equation is possible [5-29]. Thus, in the non-relativistic case, Milne-McCree solutions are obtained, a non-relativistic analogue of Friedman-type solutions of the evolution of the Universe [30-43].

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